Event-Based Formation Control of Networked Multi-Agent Systems Using Complex Laplacian Under Directed Topology

Mojtaba Ranjbar, Mohammad T.H Beheshti, Sadegh Bolouki

Abstract—**In this paper, the formation control problem of multi-agent systems with directed communication graphs is considered. A novel distributed event-triggered approach, which involves complex Laplacian, is proposed to address the problem. The event condition depends on periodic samplings of agent states, which automatically eliminates the possibility of Zeno behavior. Then, it is shown that, under simple verifiable conditions, the proposed control strategy results in the desired formation of agents. Finally, the results are verified by numerical examples.**

Index Terms—**complex Laplacian, event-based control, formation, multi-agent systems**

I. Introduction

In recent years, cooperative control of multi-agent systems has attracted many researchers due to its capability to address various problems raised within different disciplines. Cooperative control is inspired by natural phenomena such as carrying seeds by a group of ants and flying a flock of birds in V-shape. Formation control is among the most important problems of cooperative control, which has found growing applications such as target localization [1], environment monitoring [2], marine search [3]. Formation control in multi-agent systems is defined as the problem of finding the control strategy that steers the group of agents toward a desired geometrical pattern. The desired geometrical pattern may be characterized by desired distances between pairs of agents [4], [5]; desired distances between pairs of agents along with desired angles [6], [7]; or relative positions of neighboring agents [8], [9].

In implementation of cooperative control algorithms, each agent needs to be equipped with an embedded microprocessor, onboard communication unit, and actuation unit. These units have limited energy resources to carry out their tasks such as computing, transferring information to the neighboring agents, and driving the agent. In conventional cooperative control algorithms, each agent communicates with neighboring agents and updates its control actuation continuously or periodically at discrete sampling times. These time-triggered control algorithms may not be desirable and practical due to their excessive energy consumption. To address this issue, researchers have proposed event-based control schemes to replace time-based control scheme. This control policy reduces triggering frequency of electronic and actuation components which leads to reduction in energy consumption as well as prolonging life time of electronic components.

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A. Related Work

Various algorithms, based on different approaches, have been proposed in the literature to address formation control problems. In the case where the desired formation is specified by inter-agent distances, gradient control laws are most commonly used [4], [5], [10], while for formations represented by inter-agent distances and angles, bearing measurements are used to achieve the desired formation [6], [7]. With regard to formations characterized by relative positions of neighbors, consensus-based approaches are widely used [8], [9], [11].

Most related to our work, the complex Laplacian technique and event-based control have been utilized for the formation control problem. The complex Laplacian-based approach has recently been introduced to obtain the planar formation [12]–[14]. Main features of this approach can be listed as the following: (i) it allows four degrees of freedom in the formation, which are rotation, scaling, and two-dimensional translation, (ii) a common coordinate system is no longer required as opposed to consensus-based formation control schemes, and (iii) unlike gradient control laws, complex Laplacian-based approach leads to linear control protocols, which leads to the *global* asymptotic stability of the system.

Event-based control is introduced as a replacement to time triggered control as it provides better resource management and communication load reduction. Various articles exist that address event-based consensus control [15]–[18] although very few works undertake the formation control problem. For instance, in [19], a dynamic event-based communication mechanism to adjust inter-agent communication and distributed event-based formation control is introduced. The control algorithm is consensus-based and uses locally triggered sampled-data. However, the dynamic event communication mechanism requires all sampled state information to adjust the dynamically varying threshold parameter in the event triggering condition. As another example, authors in [20] consider a network of first-order multi-agent system with a weight-unbalanced, directed communication graph and investigate distributed asynchronous event-based communication and control for the circle formation problem. An event-based time-varying formation control for a general linear multi-agent system over undirected network topology is introduced in [21], where the approach involves an event condition for each agent depending on the agent's own state only. Centralized and decentralized event-based control laws for solving circle formation problem in the second-order dynamics are proposed in [22]. Event-triggered formation control based on complex-valued Laplacian for both continuous and discrete time dynamics is considered in [23], where agents interact with each other over an undirected graph and the triggering condition varies over time. The sampled event-triggered mechanism has various advantages compared with continuous event-triggered mechanism addressed in [20]–[22] listed in the following: (i) unlike continuous event-triggered mechanism which require to check event condition continuously, sampled event-triggered mechanism evaluate event condition only at periodic sampling time, (ii) this strategy is compatible for implementation of digital control platform, and (iii) there exists a minimum inter-event interval lower-bounded by the sampling period of event condition, which reduces the communication load and save energy resources for detect on events.

B. Our Contributions

We propose in this work an event-based formation control using complex Laplacian for a network with directed communication topology. Our event detector, which triggers communications between agents, is based on the Lyapunov approach. It is distributed and uses complex and sampled information. We obtain parameters of event detector such that the time derivative of Lyapunov function is made negative semi-definite. We also show minimum inter-event intervals are lower-bounded by a positive value.

Main contributions of this paper are as follows. First, a distributed event-based communication mechanism based on Lyapunov approach is extracted in Section III and event-based formation control problem using complex Laplacian approach under directed topology is solved. Secondly, necessary and sufficient conditions for convergence of the algorithm is obtained in Section IV, where the Zeno behavior is also evaluated and it is proven that chattering phenomenon is excluded. The efficacy of the proposed algorithm is demonstrated in Section V. While the complex Laplacian technique for the event-based formation control has previously been employed given networks with undirected topologies [23], it has not been extended to networks with directed topologies, which constitutes our focus in this work. The extension is in no way straightforward and requires major modifications detailed in Sections III and IV.

II. PRELIMINARIES

A. Graph Theory

The communication topology of a network of agents is described by a directed graph $G = \{v, \varepsilon\}$ that consists of a set $v = \{v_1, v_2, \ldots, v_n\}$ of nodes and a set $\varepsilon \subset v \times v$ of edges. A generic edge e_{ji}, alternatively denoted by (v_j, v_i), indicates existence of an information link from agent v_j to agent v_i, which means agent v_i, referred to as the child agent, receives information from v_j, referred to as the parent agent. The set of in-neighbors, or simply neighbors, of an agent i is denoted by N_i. Thus, $N_i = \{v_j \in v : (v_j, v_i) \in \varepsilon\}$. Given a subset u of nodes, a node ϑ is said to be *reachable* from u if there exists a path from a node in u to ϑ. Moreover, ϑ is said to be *2-reachable* from set u if there exists a path from a node in u to ϑ after removing any one node except node ϑ from u. In other words, there have to exist two disjoint paths from u to ϑ. A digraph G is said to be *2-rooted* if there exists a subset of two nodes from which every other node is 2-reachable. These two nodes are referred to as *roots* of the digraph. The complex weights $w_{ij} \neq 0$ are assigned to edges e_{ji}. The definition of complex Laplacian of a digraph is stated in the following.

Definition 1: Given a digraph $G = \{v, \varepsilon\}$, its *complex Laplacian* $L = [l_{ij}] \in \mathbb{C}^{n \times n}$ is defined as

$$l_{ij} = \begin{cases} -w_{ij} & \text{if } i \neq j \text{ and } j \in N_i \\ 0 & \text{if } i \neq j \text{ and } j \notin N_i \\ \sum\limits_{k \in N_i} w_{ik} & \text{if } i = j \end{cases}$$

It is to be noted that elements of the complex Laplacian, as opposed to those of the real Laplacian, can be complex numbers. Given a complex square matrix A, the spectral norm of A denoted by $\|A\|_2$ is the largest singular value of A, that is the square root of the largest eigenvalue of the matrix A^*A where A^* indicates the conjugate transpose of A. For the sake of notational simplicity, we write $\|A\|$ herein to refer to the spectral norm $\|A\|_2$. The positive semi-definite concept can be extended to arbitrary complex matrix A, complex matrix A is called positive semi-definite if $Re[x^*Ax] \geq 0$ where $x \in \mathbb{C}^n$ is any nonzero complex vector and $Re(c)$ indicates real part of complex number c. A necessary and sufficient condition for a matrix A to be positive semi-definite is that the hermitian part $A_H = (\frac{A+A^*}{2})$, be positive semi-definite.

B. Planar Formation

A 2-D *geometric formation* in the complex plane is represented by a vector $\xi = [\xi_1, \xi_2, \ldots \xi_n]^{\mathrm{T}}$, comprising of n complex values, called formation basis. Throughout this paper, geometric formation is considered to be generic in the sense that the values $\xi_1, \ldots \xi_n$ do not satisfy any nontrivial algebraic equation with integer coefficients [24]. Given a formation ξ, a *similar formation* of ξ is characterized as $F_\xi = c_1 1_n + c_2 \xi$. where c_1 and c_2 are complex numbers. We notice that c_1 indicates a two-dimensional translation, while c_2 corresponds to operation of rotation and scaling. For the directed graph G with complex Laplacian L, a similar formation of shape ξ is realizable if the null space of L satisfies [13].

$$Ker\,(L) = \{c_1 1_n + c_2 \xi \ : \ c_1, c_2 \in \mathbb{C}\}\,. \tag{1}$$

Equation (1) means that the complex Laplacian L has only two zero eigenvalues which correspond to two linearly independent eigenvectors 1_n and ξ. For (1) to hold, we must have

$$rank\,(L) = n - 2, \;\; L1_n = 0 \text{ and } L\xi = 0.$$

A well-known property of the Laplacian matrix, $L1_n = 0$ always holds. If formation of shape ξ is realizable, $L\xi = 0$ can be fulfilled by selecting complex weights w_{ij} such that

$$\sum_{j \in N_i} w_{ij}\,(\xi_j - \xi_i) = 0, \;\; i = 1, 2, \ldots, n. \tag{2}$$

As mentioned in [14], if directed graph G had 2-rooted connectivity, similar formation of shape ξ is realizable.

Lemma 1: [14] For a generic formation basis ξ on directed graph G, a similar formation of shape ξ is realizable if and only if G is 2-rooted.

Noted that, result on connectivity of graph is based on the assumption that geometric formation to be generic.

III. Event-Triggered Strategy

In this section, we first characterize the distributed sampled event detector, which is used to trigger the control input described afterwards.

A. Distributed Event-Triggered Communication Mechanism

Consider a group of agents exchanging information within a directed network with fixed topology. The single-integrator kinematics of each agent is considered as

$$\dot{z}_i = u_i\left(t\right),\ 1 \le i \le n, \tag{3}$$

where $z_i \in \mathbb{C}$ is the position of agent i and $u_i\left(t\right) \in \mathbb{C}$ is the velocity control input. An event-triggering scheme introduced in [16] for solving consensus problem has inspired us to propose sampled event triggered communication mechanism to reduce communication load and save energy resources. Each agent has sampled event detector that receives the states of the agent and its neighbors to determine when control inputs should be updated. The proposed distributed sampled event detector, which only uses local information, employs the following event condition:

$$\left\|e_i(t_r^i + lT)\right\|^2 \le \sigma_i \left\|y_i(t_r^i + lT)\right\|^2,\ l = 1, 2, \ldots \tag{4}$$

where t_0^i is set at 0 and the parameter σ_i is positive whose value will be determined later on. The constant T is the sampling period, which is uniform among all agents. t_r^i is the rth event time of agent v_i and is an integer multiple of the sampling period T. The measurement error $e_i(t_r^i + lT)$ is the difference between the state value at the last event and the state value at the current sampling time. This error has a crucial role in the event-based control scheme. When the norm of the measurement error reaches a state dependent threshold, an event is occurred and, consequently, the control signal is updated. The measurement error of ith agent is defined as

$$e_i(t_r^i + lT) = z_i(t_r^i) - z_i(t_r^i + lT). \tag{5}$$

In the event condition (4), the endogenously driven threshold $y_i(t_r^i + lT)$ is considered as

$$y_i(t_r^i + lT) = \sum_{j \in N_i} m_{ij}\left(z_i(t_r^i + lT) - z_j(t_r^i + lT)\right),$$

where m_{ij}'s are elements of the matrix M defined as

$$M = \frac{F + F^*}{2} \text{ and } F = L^* D^* L^* D^* D L,$$

where *arbitrary stabilizing diagonal matrix* $D = \operatorname{diag}\left([d_1, \ldots, d_n]\right)$ locates unstable non-zero eigenvalues of closed loop matrix DL within the left half-plane [12]. Unlike real Laplacian matrices, whose eigenvalues all lie within the closed right half of the complex plane, complex-valued Laplacian matrices may possess left half-plane eigenvalues. According to the following lemma from [12], however, using a stabilizing diagonal matrix D, all but two eigenvalues of DL, which are both at zero, can be placed at any desired values in the complex plane.

Lemma 2 ([12]): Let a 2-rooted graph G and a formation basis ξ be given. Then, for almost any Laplacian L of G satisfying $L\xi = 0$, using a proper stabilizing matrix D, all eigenvalues of $-DL$, except two at zero, can be placed at any desired values in the complex plane.

Each agent requires its own state along with states of its neighbors at discrete sampling times to check whether the event condition (4) is satisfied. Once the event condition (4) is violated, an event is triggered. At an event time, the control input of agent is locally updated by triggered sampled state information of the agent. Moreover, triggered sampled state information is sent to the agent's neighbors so that they also update their control action given the new information. Furthermore, the measurement error $e_i(t_r^i + lT)$ is set to zero immediately and the inequality (4) is subsequently satisfied.

$$t_{r+1}^i = t_r^i + T \inf\left\{l : \left\|e_i(t_r^i + lT)\right\|^2 > \sigma_i \left\|y_i(t_r^i + lT)\right\|^2\right\} \tag{6}$$

event instants of agent v_i are integer multiple of sampling period T, in other words $\left\{t_0^i, t_1^i, \ldots\right\} \subseteq \{0, T, 2T, \ldots\}$ this implies minimum inter-event intervals are lower-bounded by a sampling period T, so this guarantes the Zeno behavior is excluded.

B. Distributed Event Triggered Formation Control

The event-based control signal of each agent is considered as

$$u_i(t) = -d_i \sum_{j \in N_i} w_{ij}\left(\hat{z}_i\left(t\right) - \hat{z}_j\left(t\right)\right), \tag{7}$$

where $\hat{z}_i\left(t\right) \triangleq z_i\left(t_r^i\right)$ for the unique r satisfying

$$t_r^i \le t < t_{r+1}^i.$$

$\hat{z}_j$ is defined in a similar fashion.

IV. Convergence Analysis

We now determine conditions under which our proposed event-based control steers agents' states to converge to a similar formation of the desired shape. Applying the control law (7) to the first-order system (3), the dynamics of the closed loop system can be described as

$$\dot{z}_i = -d_i \sum_{j \in N_i} w_{ij}\left(\hat{z}_i\left(t\right) - \hat{z}_j\left(t\right)\right). \tag{8}$$

By some calculations, given $t \in \left[t_r^i + lT\,,\, t_r^i + lT + T\right)$, the closed-loop dynamics (8) can be rewritten as

$$\begin{aligned}
\dot{z}_i &= -d_i \sum_{j \in N_i} w_{ij}\left(z_i\left(t_r^i\right) - z_j\left(t_{r'}^j\right)\right) \\
&= -d_i \sum_{j \in N_i} w_{ij}\left(z_i\left(t_r^i + lT\right) - z_j\left(t_r^i + lT\right)\right) \\
&\quad - d_i \sum_{j \in N_i} w_{ij}\left(z_i\left(t_r^i\right) - z_i\left(t_r^i + lT\right)\right) \\
&\quad + d_i \sum_{j \in N_i} w_{ij}\left(z_j\left(t_{r'}^j\right) - z_j\left(t_r^i + lT\right)\right)
\end{aligned}$$

where $t_{r'}^{j}$ is the latest event time of neighboring agent v_j, that is

$$t_{r'}^{j}=\max\left\{t\,|\,t\in\left\{t_r^j\ r=0,1,\ldots\right\},\ t\leq t_r^i+lT\right\}.$$

We notice that, $z_j\left(t_{r'}^{j}\right)$ is constant and is equal to $z_j\left(t_r^i\right)$ at the consecutive sampling times $t\in\left[t_r^i+lT\,,\,t_r^i+lT+T\right)$. Thus, the closed loop dynamics can be obtained as

$$\begin{aligned}\dot{z}_i=&-d_i\sum_{j\in N_i}w_{ij}\left(z_i\left(t_r^i+lT\right)-z_j\left(t_r^i+lT\right)\right)\\&-d_i\sum_{j\in N_i}w_{ij}\left(e_i\left(t_r^i+lT\right)-e_j\left(t_r^i+lT\right)\right),\end{aligned}$$

As it can be seen from control law (7), each agent updates its control input not only at its own event times, but also at those of its neighbors. This means that the control input of each agent is not constant between its consecutive event times. However, the control input of an agent is still constant between any two consecutive sampling times. Thus, the closed loop dynamics of the system, given $t\in[kT,\ (k+1)\,T)$, can be derived as

$$\dot{z}\left(t\right)=-DLz\left(kT\right)-DLe\left(kT\right),\tag{9}$$

where $z=[z_1,\ldots,z_n]^T$ and $e=[e_1,\ldots,e_n]^T$. From (9), $z\left(t\right)$, for $t\in[kT,\ (k+1)\,T)$, is solved as

$$z\left(t\right)=z\left(kT\right)-\left(t-kT\right)DL\left(z\left(kT\right)+e\left(kT\right)\right).$$

We now examine the stability of closed loop dynamics (9) by introducing a particular quadratic Lyapunov function $V(t)$ as the following:

$$V\left(t\right)=\frac{1}{2}z^*\left(t\right)L^*D^*DLz\left(t\right).\tag{10}$$

To prove stability, it suffices to show that the Lyapunov function $V(t)=\frac{1}{2}(\|DLz\left(t\right)\|)^2$ vanishes as t grows, in which case $z(t)$ converges to $c_1 1_n+c_2\xi$, meaning the desired formation is obtained. Computing time derivative of $V(t)$ along the trajectories of closed loop system (9) for $t\in[kT,\ (k+1)\,T)$, we have

$$\dot{V}\left(t\right)=\frac{1}{2}\dot{z}^*\left(t\right)L^*D^*DLz\left(t\right)+\frac{1}{2}z^*\left(t\right)L^*D^*DL\dot{z}\left(t\right),$$

which then yields

$$\begin{aligned}\dot{V}\left(t\right)=&-\frac{1}{2}\Big(\left(z^*\left(kT\right)+e^*\left(kT\right)\right)L^*D^*\Big)L^*D^*DL\\&\times\Big(z\left(kT\right)-\left(t-kT\right)DL\big(z\left(kT\right)+e\left(kT\right)\big)\Big)\\&-\frac{1}{2}\Big(z^*\left(kT\right)-\left(t-kT\right)\left(z^*\left(kT\right)+e^*\left(kT\right)\right)L^*D^*\Big)\\&\times L^*D^*DLDL\big(z\left(kT\right)+e\left(kT\right)\big).\end{aligned}$$

We now define $F\triangleq L^*D^*L^*D^*DL$, and $F_1\triangleq FDL$. We notice that both matrices F and F_1 are positive semi-definite matrices with same zero eigenvalues, while F_1 is also a hermitian matrix, i.e., $F_1=F_1^*$. We thus conclude that

$$\begin{aligned}&\dot{V}\left(t\right)=-z^*\left(kT\right)\left(\frac{F+F^*}{2}\right)z\left(kT\right)+\left(t-kT\right)z^*\left(kT\right)F_1z\left(kT\right)\\&+\left(t-kT\right)z^*\left(kT\right)F_1e\left(kT\right)+\left(t-kT\right)e^*\left(kT\right)F_1z\left(kT\right)\\&-\frac{1}{2}e^*\left(kT\right)Fz\left(kT\right)-\frac{1}{2}z^*\left(kT\right)F^*e\left(kT\right)+\left(t-kT\right)e^*\left(kT\right)F_1e\left(kT\right)\\&\leq-z^*\left(kT\right)\left(\frac{F+F^*}{2}-TF_1\right)z\left(kT\right)+Te^*\left(kT\right)F_1e\left(kT\right)+\\&|e^*\left(kT\right)\left(-\frac{1}{2}F+\left(t-kT\right)F_1\right)z\left(kT\right)\\&+z^*\left(kT\right)\left(-\frac{1}{2}F^*+\left(t-kT\right)F_1\right)e\left(kT\right)|\end{aligned}$$

We consider the value $(t-kT)$ has its maximum value which is equal to T at the consecutive sampling times $t\in[kT,\ (k+1)\,T)$. Matrix $J=-\frac{1}{2}F+TF_1$ is negative semi-definite if the hermitian matrix $(\frac{J+J^*}{2})$ be negative semi-definite. By selecting the sampling period T, with $T\leq\frac{\lambda_1}{2\lambda_{F_1}}$, matrix $(\frac{J+J^*}{2})$ and consequently matrix J will be made negative semi-definite. The value λ_1 is the smallest non-zero eigenvalue of the matrix $M=\left(\frac{F+F^*}{2}\right)$ and λ_{F_1} is maximum eigenvalue of the matrix F_1. We notice that, all eigenvalues of hermitian matrices M and F_1 are real. Following inequality the multiplication of error into state can be separated:

$$\begin{aligned}&e^*\left(kT\right)\left(-\frac{1}{2}F+TF_1\right)z\left(kT\right)+z^*\left(kT\right)\left(-\frac{1}{2}F^*+TF_1\right)e\left(kT\right)\leq\\&z^*\left(kT\right)\left(\frac{F+F^*}{4}-TF_1\right)z\left(kT\right)+e^*\left(kT\right)\left(\frac{F+F^*}{4}-TF_1\right)e\left(kT\right)\end{aligned}$$

Thus, we can write the upper bound of time derivative of Lyapunov function as the following:

$$\begin{aligned}\dot{V}\left(t\right)\leq&-\frac{1}{2}z^*\left(kT\right)\left(\frac{F+F^*}{2}\right)z\left(kT\right)\\&+\frac{1}{2}e^*\left(kT\right)\left(\frac{F+F^*}{2}\right)e\left(kT\right)\\\leq&-\frac{1}{2}z^*\left(kT\right)\left(\frac{F+F^*}{2}\right)z\left(kT\right)\\&+\frac{1}{2}\lambda_ne^*\left(kT\right)e\left(kT\right),\end{aligned}\tag{11}$$

where λ_n is maximum eigenvalue of the matrix M. The event condition (4) can now be written in the following compact form:

$$\begin{aligned}&e^*\left(kT\right)e\left(kT\right)\leq\sigma_{\max}Y^*\left(kT\right)Y\left(kT\right)\\&=\sigma_{\max}z^*\left(kT\right)M^*Mz\left(kT\right)=\sigma_{\max}\lambda_nz^*\left(kT\right)Mz\left(kT\right),\end{aligned}\tag{12}$$

where $e\left(kT\right)=\left[e_1\left(kT\right),\ldots,e_n\left(kT\right)\right]^T$ and $Y(kT)=\left[Y_1\left(kT\right),\ldots,Y_n\left(kT\right)\right]^T$. Incorporating (12) into (11) implies that

$$\begin{aligned}\dot{V}\left(t\right)\leq&-\frac{1}{2}z^*\left(kT\right)\left(\frac{F+F^*}{2}\right)z\left(kT\right)\\&+\frac{1}{2}{\lambda_n}^2\sigma_{\max}z^*\left(kT\right)\left(\frac{F+F^*}{2}\right)z\left(kT\right)\\=&-\frac{1}{2}\left(1-{\lambda_n}^2\sigma_{\max}\right)z^*\left(kT\right)\left(\frac{F+F^*}{2}\right)z\left(kT\right),\end{aligned}$$

where $\sigma_{\max} = \max\{\sigma_i \; i = 1, \ldots, n\}$. If $0 < \sigma_{\max} < \frac{1}{\lambda_n{}^2}$, then $\dot{V}(t) \le 0$. Now, consider the invariant set Ω defined as

$$\Omega \triangleq \left\{ z \in \mathbb{C}^n \mid \dot{V}(t) \le c \right\},$$

where c is an positive arbitrary small value.

Since the graph G is 2-rooted, the set Ω is bounded and closed. Invoking LaSalle invariance principle, we can conclude that each solution starting from Ω will converge to the largest invariant set in $S = \left\{ z \in \Omega \mid \dot{V}(t) = 0 \right\}$. Now we aim to obtain the invariant set S in which $\dot{V}(t) = 0$. In order to have $\dot{V}(t) = 0$, it is required $z^*(kT)\left(\frac{F+F^*}{2}\right)z(kT) = 0$. Since $\frac{F+F^*}{2}$ is positive semi-definite, we can write $\left\| \left(\frac{F+F^*}{2}\right)^{\frac{1}{2}} z(kT) \right\| = 0$. Therefore, $\left(\frac{F+F^*}{2}\right)z(kT) = 0$. Due to lemma from [25], the null-space of $\left(\frac{F+F^*}{2}\right)$, which is positive semi-definite hermitian part of F, is equal to null-space of F^*. Thus, we must have $F^* z(kT) = 0$ or equivalently,

$$L^* D^* DLDLz(kT) = 0. \tag{13}$$

Due to another theorem sated in [25], the null-space of $L^* D^* DL$ is equal to the null-space of DL. Therefore, (13) implies that $(DL)^2 z(kT) = 0$, and consequently, $DLz(kT) = 0$. Hence, the invariant set S is identical to $Ker(L)$.

$$S = Ker(L) = \{c_1 1_n + c_2 \xi : c_1, c_2 \in \mathbb{C}\}.$$

This means that a similar formation of shape ξ is obtained. Based on the discussion above, the following theorem can be expressed.

Theorem 1: Consider a network of agents with dynamics (3), where the communication graph is directed and 2-rooted. Applying the control law (7), driven by the event condition (4), to the system (3), a similar formation shape ξ will be reached if the below constraints on parameters of the event detector are satisfied:

$$0 < T \le \frac{\lambda_1}{2\lambda_{F_1}}; \text{ and } 0 < \sigma_{\max} < \frac{1}{\lambda_n{}^2}.$$

Remark 1: To determine sampling period T and threshold parameters $\sigma_i \le \sigma_{\max} < \frac{1}{\lambda_n{}^2}$ $1 \le i \le n$ which specify data transmission rates, it is required to assume the global knowledge of the complex Laplacian L.

V. Numerical Results

In this section, the performance of the proposed event-based algorithm for formation control of first order multi-agent systems is confirmed via simulations. We consider a network of agents from [26], whose directed communication graph is depicted in Fig. 1. One can easily verify, according to the following, that the communication topology is 2-rooted: if nodes 1,4 are considered as the subset of two roots of digraph, every other node is 2-reachable from this set. Let $\xi = [2i, -1+i, -1-i, -2i, 1-i, 1+i]^T$ be the desired formation shape. Moreover, the stabilizing matrix $D = \text{diag}([-0.3, 0.3, 0.4, 0.4, 0.3, 0.25])$ is selected, which places all but two eigenvalues of $-DL$ in the left half-plane. Arbitrary complex weights w_{ij}'s are given such that $L\xi = 0$.

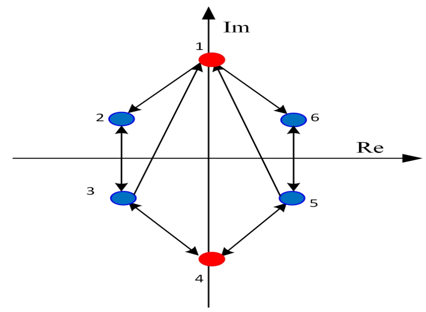

Fig. 1: Formation shape and communication topology of six agents.

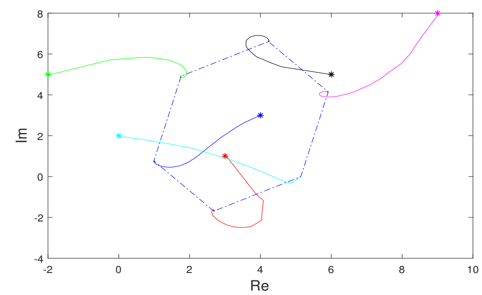

Fig. 2: Trajectory of agents.

$$L = \begin{bmatrix} -2 & 0 & 1-3i & 0 & 1+3i & 0 \\ -2 & 3-i & -1+i & 0 & 0 & 0 \\ 0 & -1 & 2-i & -1+i & 0 & 0 \\ 0 & 0 & -1 & 1-i & i & 0 \\ 0 & 0 & 0 & -2 & 3-i & -1+i \\ -2 & 0 & 0 & 0 & -1-i & 3+i \end{bmatrix}.$$

The parameters of event detector and sampling period should be chosen such that constraints of Theorem 1 are satisfied, which translate to

$$0 < T \le 0.0292s \text{ and } 0 < \sigma_{\max} < 0.0529.$$

For instance, the parameters of event detector and sampling period can be selected as $\sigma_1 = 0.05$, $\sigma_2 = 0.04$, $\sigma_3 = 0.03$, $\sigma_4 = 0.04$, $\sigma_5 = 0.05$, $\sigma_6 = 0.02$, and $T = .01s$. We now apply the event-based control law (7) to the multi-agents system (3) under event condition (4). Figure 2 shows The evolution of agents in the complex plane. It can be seen that a similar formation of shape ξ can be achieved. The evolution of the error signal $\|e_1(kT)\|$ is depicted in Fig. 3 for instance. We notice that when an event is triggered, the measurement error $e_1(kT)$ is set to zero immediately.

Figure 4 shows that signal $Lz(kT)$ converges to zero asymptotically. This means that the trajectories of the system converge to a similar formation shape ξ.

The event times of all agents are depicted in Fig. 5 which illustrates that the number of information exchanges between neighboring agents is decreased significantly compared to continuous time communication mechanism.

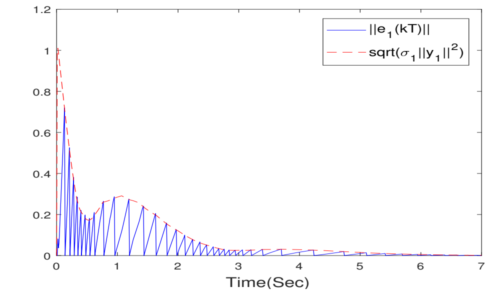

Fig. 3: Error signals and threshold function of agent 1.

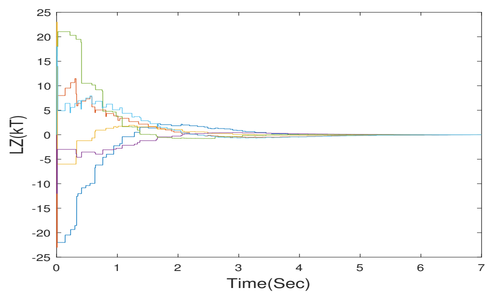

Fig. 4: The evolution of $Lz(kT)$ at triggered sampled states

VI. Conclusion

In this paper, the event-based formation control of multi-agent systems using complex Laplacian technique was considered. A distributed sampled event detector using Lyapunov approach was proposed such that event detector determines when agents update their control signals. We also ran simulations in order to evaluate the performance of the proposed control law. Simulation results showed proposed control law has a better performance with respect to the time driven formation control algorithm due to a significant reduction of communication load and saving energy resource.

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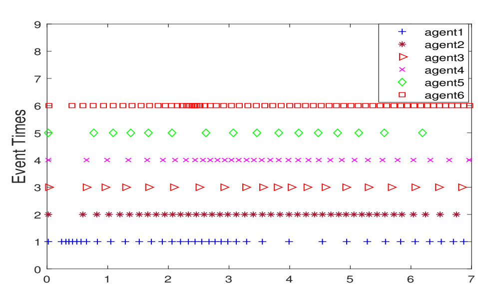

Fig. 5: Event times of each agent.

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